Warped product Ricci solitons and warping function estimates.

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Abstract

This talk aims to investigate *warped product gradient Ricci solitons*, namely, gradient Ricci solitons isometric to a warped product $M^n \times_h F^m$, where M and F are the base and the fiber of the warped product, respectively, and $h : M \to \mathbb{R}$ is the positive smooth warping function. Ricci solitons with this geometry arise naturally when requiring its Weyl tensor to be harmonic [6].

When h is not constant and M is complete, it was shown in [2, Corollary 2.2] that the potential function $f : M \times F \to \mathbb{R}$ of the soliton is lifted from M, and that F^m is Einstein. This reformulates the investigation solely in terms of equations on the base M and the sign the Einstein constant of F, say μ, a point of view adopted in [1, 3, 5], for example.

In the talk, we present several estimates to h and $|\nabla(\ln h)|^2$, which assume different forms according to the signs of λ and μ. They are proved with the aid of different types of maximum principles. We also present a rigidity result for the shrinking ones, based on the study of the first eigenvalue $\lambda_1(-\Delta_{f-m\ln h})$ of a weighted Laplacian, following an approach inspired by a classical work of Fujita [4]. These results can be found in [1].

References

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